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Funciones t-migrativas t-overlap: una generalización de migrtividad en funciones t-overlap

T-overlap T-migrative Functions: a generalization of migrativity in t-overlap functions

Funções T-sobrepostas T-sobrepostas: uma generalização da migração para funções T-sobrepostas

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Resumen.

Este artículo introduce una generalización de funciones migrativas por extensión de la condición de la operación producto aplicada en las variables. Más específicamente, en lugar de exigir multiplicar la variable x por un número real α, en este trabajo se trabaja este número α con las variables de acuerdo a una t-norma. Se denomina a esta generalización función t-migrativa con respecto a tal t-norma. Luego se analizan las propiedades principales de funciones t-migrativas en funciones t-overlap y se introducen algunos métodos de construcción de este tipo de funciones.

Palabras clave: función migrativa, función overlap, normas triangulares.

Abstract

This paper introduces a generalization of migrative functions by extending the conditions of the product operation applied in the variables. More specifically, instead of requiring to multiply the variable x by a real number α, in this work we operate this α number with the variables according to a t-norm. We call such generalization as a t-migrative function with respect to such t-norm. Then we analyze the main properties of t-migrative t-overlap functions and introduce some construction methods.

Keywords: migrative function, overlap function, t-norm.

Resumo

Este artigo apresenta uma generalização das funções mi-

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gratórias estendendo as condições da operação do produto aplicada nas varivéis. Mais especificamente, em vez de exigir a multiplicação da varivél x por um número real α, neste trabalho operamos isso α número com as variáveis de acordo com a norma t. Chamamos essa generalização de uma função t-migrative em relação a essa norma t. Em seguida se analiza as principais propriedades das funções t-sobreposição migratórias e introduzimos alguns métodos de construoção.
Palavras-chaves: função migratória, função de sobreposição, norma t .

Introduction

The notion of migrative function was introduced in [33]. Our goal here is to generalize the notion of migrative functions by relaxing one of the conditions, in particular, instead of demanding the variables of the function to be multiplied with a real number α. In this work, we replaced the multiplication by a t-norm. We allow it some kind of threshold, defined in terms of a t-norm T. We call such generalization as a t-migrative function with respect to T after this study we apply the migrativity to t-overlap functions. We notice that, this simple generalization allows us to state several interesting properties, which may allow for application in fuzzy rule-based system in order to discard bad rules when computing the compatibility degree. Section 1 presents some preliminary concepts. In Sect. 2, besides studying the main properties, we also propose some construction methods.

1. Preliminares

1.1. Triangular Norms

One of the basic concepts of the fuzzy theory study is about triangular norms or t-norms. In this work, these functions will also be used very frequently, the definition of a t-norm is anunciated below.

Definition 1 *(See [30]) A triangular norm or t-norm is an aggregation function*
$T : [0,1]^2 \to [0,1]$ *that follows the next.properties:*

(i) $T(x,1) = x$ *for all* $x \in [0,1]$,

(ii) $T(x,y) \leq T(z,u)$ *if* $x \leq z$ *and* $y \leq u$,

(iii) $T(x,y) = T(y,x)$ *for all* $x,y \in [0,1]$,

(iv) $T(T(x,y),z) = T(x,T(y,z))$ *for all* $x,y,z \in [0,1]$.

Below are some examples of t-norms, which are widely known

Example 1 *The function* $T : [0,1]^2 \to [0,1]$, *defined by* $T(x,y) = \mathrm{m\ n}\{x,y\}$ *is a t-norm.*

Definition 2 *It says that a* $t{-}norm$ *is strict, if it is strictly increasing for its two variables, that is to say if* $x_1 < x_2$ *and* $y \neq 0$ *then* $T(x_1,y) < T(x_2,y)$.

Definition 3 *It It says that a* $t - norm$ *is positive, if it is true that* $T(x,y) = 0$ *iff* $xy = 0$.

1.2. Migrativity

The concept of α-migrativity was introduced by Durante et al. in [33] a bivariate operation's class having a property previously presented by Mesiar and Novak in 28, as Fodor and Rudas acknowledge in 32.

Definition 4 *(See [33]) Let* $\alpha \in [0,1]$ *be fixed. A bivariate operation*
$G : [0,1]^2 \to [0,1]$ *is* α*-migrative if* $G(\alpha x, y) = G(x, \alpha y)$, *for all* $x,y \in [0,1]$.

It is easy to see this definition is that all function $G : [0,1]^2 \to [0,1]$ is 1-migrative, as $G(x,y) = G(1.x,y) = G(x,1.y)$. This definition is referred a predetermined α. The concept of α-migrativity has been generalized from the next form,

Definition 5 *(See [34]) A function* $G : [0,1]^2 \to [0,1]$ *is called migrative if and only if* $G(\alpha x, y) = G(x, \alpha y)$ *for all* $x,y \in [0,1]$ *and for all* $\alpha \in [0,1]$.

Example 2 *The function* $h : [0,1]^2 :\to [0,1]$, *defined by* $h(x,y) = xy$, *is migrative.*

Example 3 *La function* $G : [0,1]^2 \to [0,1]$, *defined by* $G(x,y) = \frac{x+y}{2}$, *shows that*

$$\begin{aligned} G(1.x,y) &= \frac{1.x+y}{2} \\ &= \frac{x+1.y}{2} \\ &= G(x,1.y) \end{aligned}$$

thus G *is a 1-migrative function, but if you take* $\alpha = \frac{1}{2}$, *it shows that* $G(\frac{1}{2}x, y) \neq G(x, \frac{1}{2}y)$. *This is easy to see, in particular if it is done* $x = 1$ *and* $y = \frac{1}{2}$.

The following is a reminder of one of the main migrative functions' characterizations.

Lemma 1 *(See [1]) A function*

$$G : [0,1]^2 \to [0,1]$$

is migrative if and only if $G(x,y) = G(1,xy)$, for all $x,y \in [0,1]$.

By the lemma [5], you get the following corollary, in which a characterization of bivariate migrative function is seen as a function of a variable.

Corollary 1 *(See [1]) A function $G : [0,1]^2 \to [0,1]$ is migrative if and only if exists a function $g : [0,1] \to [0,1]$ such that $G(x,y) = g(xy)$ for all $x,y \in [0,1]$.*

1.3. Overlap and T-Overlap Functions

The definition of overlap function is stated as well as some of its properties. This type of functions constitute one of the most important pillars in this work.

Definition 6 *(See [1]) A function $G_S : [0,1]^2 \to [0,1]$ is an overlap function, if it follows the next conditions:*

(G_s1) *G_s is symmetrical,*

(G_s2) *$G_s(x,y) = 0$ if and only if $xy = 0$,*

(G_s3) *$G_s(x,y) = 1$ if and only if $xy = 1$,*

(G_s4) *G_s is not decreasing,*

(G_s5) *G_s is continuous.*

Example 4 *An example of overlap function is the product function $h(x,y) = xy$, with $x,y \in [0,1]$.*

Example 5 *The function $G(x,y) = \sin(\frac{\pi}{2}xy)$ is a overlap function.*

Example 6 *Other example of overlap function is $G(x,y) = \tan(\frac{\pi}{4}xy)$.*

A generalization of the overlap function concept, is obtained changing the condition (G_s2) given in 1. This property needs that, given by an overlap function G_S, $G_S(x,y) = 0 \Leftrightarrow xy = 0$. In this generalization, the product operation is replaced by a t-norm
$T : [0,1]^2 \to [0,1]$.

Definition 7 *Let $T : [0,1]^2 \to [0,1]$ be a t-norm. A function $G_T : [0,1]^2 \to [0,1]$ it says be a t-overlap function with respect to T, it holds the following conditions:*

(G_T1) $G_T(x,y) = G_T(y,x)$,

(G_T2) $G_T(x,y) = 0 \Leftrightarrow T(x,y) = 0$,

(G_T3) $G_T(x,y) = 1 \Leftrightarrow x = y = 1$,

(G_T4) *G_T is increasing,*

(G_T5) *G_T is continuous.*

2. T-Migrativity

In this section a generalization of the migrativity concept is made where the multiplication operation is replaced by a t-norm.

Definition 8 *It is said that a two-dimensional G function is t-migrative with respect to the a t-norm T if for all $\alpha \in [0,1]$ is fulfilled that $G(x,T(\alpha,y)) = G(T(x,\alpha),y)$ for all $x,y \in [0,1]$.*

As in the traditional definition, a migrativity property is given for the particular case in which $\alpha = 0$.

Proposition 1 *A function $G : [0,1] \to [0,1]$ is 0-t-migrative if and only if $G(x,0) = G(0,y)$*

Proof 1 *If G is 0-t-migrative then $G(x,0) = G(x,T(0,y)) = G(T(x,0),y) = G(0,y)$ If $G(x,0) = G(0,y)$ then as for all t-norm $T(0,x) = 0$ then $G(x,T(0,y)) = G(T(x,0),y)$.*

The following theorem broadly generalizes theorem 1

Theorem 1 *A function $G : [0,1]^2 \to [0,1]$ is t-migrative respecting to t-norma T if and only if exists a function $g : [0,1] \to [0,1]$ such that $G(x,y) = g(T(x,y))$.*

Proof 2 *Let $G(x,y) = g(T(x,y))$, then*

$$G(x,T(y,z)) = g(T(x,T(y,z))) = g(T(T(x,y),z)) = G(T(x,y),z).$$

If G is t-migrative respecting to t-norm T, then

$$G(x,y) = G(T(x,1),y) = G(1,T(x,y))$$

for all $x,y \in [0,1]$, if $G(x,y) = G(u,v)$ then $G(1,T(x,y)) = G(1,T(u,v))$ when $T(x,y) = T(u,v)$ thus g is well defined

The next corollary is a generalization of corollary 1

Corollary 2 *G is t-migrative respecting to t-norm T if and only if*

$$G(x,y) = G(1,T(x,y))$$

The following corollary shows an obvious consequence that can be obtained from t-migrativity.

Corollary 3 *If G is t-migrative respecting to t-norm T then G is symetrical.*

Proof 3 $G(x, y) = G(1, T(x, y)) = G(1, T(y, x)) = G(y, x)$.

Below there is a list of some of the of t-migrative functions properties.

Theorem 2 *Let $G : [0, 1]^2 \to [0, 1]$ be a t-migrative function, then:*

1. *G is no decreasing if and only if g is no decreasing.*
2. *G is strictly increasing in $[0, 1]^2$ if and only if g and T are strictly increasing.*
3. *$G(1, 1) = 1$ if and only if $g(1) = 1$.*
4. *$G(0, 0)$ if and only if $g(0)$.*
5. *G is continuous if and only if g and T are continuous.*

Proof 4
1. *Suppose that G is a no decreasing function. Let $x, y \in [0, 1]$ such that $x \leq y$, then $G(x, 1) \leq G(y, 1)$ thus $g(T(x, 1)) \leq g(T(y, 1))$ then $g(x) \leq g(y)$. If it is assumed that g is a no decreasing function and $x, y \in [0, 1]$ such that $x \leq y$ then for all $z \in [0, 1]$ is true that $T(x, z) \leq T(y, z)$, therefore $g(T(x, z)) \leq g(T(y, z))$ thus $G(x, z) \leq G(y, z)$.*
2. *Analogous.*
3. *$G(1, 1) \Leftrightarrow g(T(1, 1)) = 1 \Leftrightarrow g(1) = 1$*
4. *$G(0, 0) \Leftrightarrow g(T(0, 0)) = 0 \Leftrightarrow g(0) = 0$*
5. *G is continuous if and only if g and T are continuous.*

One of the most important aspects of this work is the generalization of the migratives of overlap functions. One of the results of this generalization is shown.

Theorem 3 *If G_T is a t-overlap function respect to the continuous t-norm T, then*

$$G(x, y) = G_T(1, T(x, y))$$

is a t-overlap t-migrative function respecting to T.

Proof 5
1. *Evidently G is Symmetrical.*
2. *$G(x, y) = 0 \Leftrightarrow G_T(1, T(x, y)) = 0 \Leftrightarrow T(x, y) = 0$.*
3. *$G(x, y) = 1 \Leftrightarrow G_T(1, T(x, y)) = 1 \Leftrightarrow T(x, y) = 1 \Leftrightarrow x = y = 1$.*
4. *G is continuous.*
5. *G is no decreasing.*

$G(T(x, y), z) = G_T(1, T(T(x, y), z)) = G_T(1, T(x, T(y, z))) = G(x, T(y, z))$.

The following theorem shows that the convex sum of t-overlap t-migrative functions with respect to a continuous t-norm are also t-overlap t-migrative function.

Theorem 4 *If $\alpha_i \geq 0 \forall i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} \alpha_i = 1$, G_i are overlap functions and T is a t-norm continuous, then*

$$G(x, y) = \sum_{i=1}^{n} \alpha_i G_i(1, T(x, y))$$

is t-overlap t-migrative function respecting to T.

Proof 6
1. *G is symmetrical.*
2. *$G(x, y) = 0 \Leftrightarrow \sum_{i=1}^{n} \alpha_i G_i(1, T(x, y)) = 0 \Leftrightarrow \alpha_i G_i(1, T(x, y)) = 0$. Given that $\sum_{i=1}^{n} \alpha_i = 1$ and $\alpha_i \geq 0 \forall i = 1, 2, \ldots, n$ then exist $\alpha_k \neq 0$ thus if $\alpha_k G_k(1, T(x, y)) = 0$ then $G_k(1, T(x, y)) = 0 \Rightarrow T(x, y) = 0$. If $T(x, y) = 0$. then $G_i(1, T(x, y)) = 0$ for all $i = 1, 2, \ldots, n$ thus $G(x, y) = 0$.*
3. *$G(x, y) = 1 \Leftrightarrow \sum_{i=1}^{n} \alpha_i G_i(1, T(x, y)) = 1 \Leftrightarrow \sum_{i=1}^{n} \alpha_i G_i(1, T(x, y)) = \sum_{i=1}^{n} \alpha_i$ thus $\sum_{i=1}^{n} \alpha_i(1 - G_i(1, T(x, y))) = 0$ then $\alpha_i(1 - G_i(1, T(x, y))) = 0$ for all $i = 1, 2, \ldots, n$ since $\alpha_i \geq 0 \forall i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} \alpha_i = 1$, then exist $\alpha_k \neq 0$ thus if $\alpha_k(1 - G_k(1, T(x, y))) = 0$ then $1 - G_k(1, T(x, y)) = 0 \Rightarrow G_k(1, T(x, y)) = 1 \Rightarrow T(x, y) = 1 \Rightarrow x = y = 1$.*
4. *G is continuous.*
5. *G is no decreasing.*

Theorem 5 *If T_1 and T_2 are continuous and G_T is a overlap function, then*

$$G(x, y) = G_T(T_1(x, y), T_2(x, y))$$

is a t-overlap t-migrative function respecting to T_1 or T_2.

Proof 7
1. *G is symmetrical.*
2. *$G(x, y) = 0 \Leftrightarrow G_T(T_1(x, y), T_2(x, y)) = 0 \Leftrightarrow T_1(x, y) = 0 \vee T_2(x, y) = 0$.*
3. *$G(x, y) = 1 \Leftrightarrow G_T(T_1(x, y), T_2(x, y)) = 1 \Leftrightarrow T_1(x, y) = 1 \wedge T_2(x, y) = 1 \Leftrightarrow x = y = 1$.*
4. *G is continuous.*
5. *G is no decreasing.*

Corollary 4 *If G is the previous theorem and $T_1 = T_2$ then G is t-migrative.*

Proof 8 $G(x, T(y, z)) = G_T(T_1(x, T(y, z)), T_2(x, T(y, z))) = G_T(T_1(T(x, y), z), T_2(T(x, y), z)) = G(T(x, y), z)$.

Corollary 5 *If T is a t-norm and a strong negation n, then*

$$G(x,y) = \frac{T(x,y)}{T(x,y)+nT(x,y)}$$

is a t-overlap t-migrative function respecting to T.

Theorem 6 *If G_T is a overlap function and T is a continuous t-norm, then*

$$G(x,y) = 2^{G(1,T(x,y))} - 1$$

is a t-overlap t-migrative function.

Proof 9
1. *G is symmetrical.*
2. $G(x,y) = 0 \Leftrightarrow 2^{G(1,T(x,y))} - 1 = 0 \Leftrightarrow 2^{G(1,T(x,y))} = 1 \Leftrightarrow G(1, T(x,y)) = 0 \Leftrightarrow T(x,y) = 0$.
3. $G(x,y) = 1 \Leftrightarrow 2^{G(1,T(x,y))} - 1 = 1 \Leftrightarrow 2^{G(1,T(x,y))} = 2 \Leftrightarrow G(1, T(x,y)) = 1 \Leftrightarrow T(x,y) = 1 \Leftrightarrow x = y = 1$.
4. *G is no decreasing.*
5. *G is continuous.*

Theorem 7 *Let M be continuous, increasing such that $M(x) = 0 \Leftrightarrow x = 0$ and $M(x) = 1 \Leftrightarrow x = 1$. If G_T es a overlap function and T is a continuous t-norm, then*

$$G(x,y) = M(G_T(1, T(x,y)))$$

is a t-overlap t-migrative function.

Theorem 8 *Let M be a n-dimensional function, non decreasing, continuous such that $M(x_1, \ldots, x_n) = 0 \Leftrightarrow x_i = 0$ for some $i \in 1, \ldots, n$ y $M(x_1, \ldots, x_n) = 1 \Leftrightarrow x_i = 1$ for some $i \in 1, \ldots, n$. Then*

$$G(x,y) = M(G_1, \ldots, G_n)(1, T(x,y))$$

is a t-overlap t-migrative function if G_i is an overlap function for all $i \in 1, \ldots, n$ and continuous t-norm T.

Proof 10
1. *G is symmetrical*
2. $G(x,y) = 0 \Leftrightarrow M(G_1, \ldots, G_n)(1, T(x,y)) = 0 \Leftrightarrow M(G_1(1, T(x,y)), \ldots, G_n(1, T(x,y))) = 0 \Leftrightarrow \exists k \in 1, \ldots, n$ *such that* $G_k(1, T(x,y)) = 0 \Leftrightarrow T(x,y) = 0$.
3. $G(x,y) = 1 \Leftrightarrow M(G_1, \ldots, G_n)(1, T(x,y)) = 1 \Leftrightarrow M(G_1(1, T(x,y)), \ldots, G_n(1, T(x,y))) = 1 \Leftrightarrow \exists k \in 1, \ldots, n$ *such that* $G_k(1, T(x,y)) = 1 \Leftrightarrow T(x,y) = 1 \Leftrightarrow x = y = 1$.
4. *G is non decreasing.*
5. *G is continuous.*

Theorem 9 *Let M be a n-dimensional function, continuous such that $M(x_1, \ldots, x_n) = 0 \Leftrightarrow x_i = 0$ for some $i \in 1, \ldots, n$ y $M(x_1, \ldots, x_n) = 1 \Leftrightarrow x_i = 1$ for some $i \in 1, \ldots, n$. Then*

$$G(x,y) = M(G_1(1, T_1(x,y)), \ldots, G_n(1, T_n(x,y)))$$

is a t-overlap function respecting to some t-norm T_k where T_i are continuous t-norms and G_i are overlap functions.

Theorem 10 *A function $G_S : [0,1]^2 \to [0,1]$ is an overlap t-migrative function if and only if $G_S(x,y) = g(T(x,y))$ for all $x, y \in [0,1]$ holds for some non-decreasing function $g : [0,1] \to [0,1]$ such that $g(0) = 0$ y $g(1) = 1$.*

Proof 11 *Let G_S be an overlap t-migrative function, then exists a non decreasing g function such that $G_S(x,y) = g(T(x,y))$. Now $g(0) = g(T(0,0)) = G_S(0,0) = 0$. besides $g(1) = g(T(1,1)) = G_S(1,1) = 1$ If $G_S(x,y) = g(T(x,y))$ then $G_S(x, T(y,z)) = G_S(T(x, T(y,z))) = g(T(T(x,y), z)) = G_S(T(x,y), z)$.*

Theorem 11 *If T is a continuous t-norm and n is a strong negation then $G(x,y) = \frac{T(x,y)}{T(x,y)+nT(x,y)}$ is a t-overlap t-migrative function with respect to T.*

Proof 12 *By corollary it can be said that G is a t-overlap function with respect to the t-norm T. On the other hand, $G(x, T(y,z)) = \frac{T(x,T(y,z))}{T(x,T(y,z))+nT(x,T(y,z))} = \frac{T(T(x,y),z)}{T(T(x,y),z)+nT(T(x,y),z)} = G(T(T(x,y),z))$.*

Theorem 12 *Let $G_1, \ldots, G_n$ be t-migrative overlap functions with respect to T. If $\omega_1, \ldots, \omega_n$ are not negative real numbers such that $\sum_{i=1}^n \omega_i = 1$ then*

$$G(x,y) = \sum_{i=1}^n \omega_i G_i(x,y)$$

is a t-migrative overlap function with respect the t-norm T.

Proposition 2 *If G_T is a t-overlap function with respect to t-norm T, then G_T is t-migrative with respect to T.*

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